A STUDY OF ALTERNATIVE QUANTILE ESTIMATION METHODS IN NEWSBOY-TYPE PROBLEMS

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THESIS

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QUANTILE ESTIMATION METHODS
IN NEWSBOY-TYPE PROBLEMS

by

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March 1980

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The newsboy problem solutions under the conditions of risk and of uncertainty about demand are well known. The former is the case where the distribution of demand is known or estimated, and the latter is the case where it is not known but the range of demand is given.

Minimizing the expected cost under risk and the minimax approach under uncertainty are well-known methods to solve the problem. The

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A Study of Alternative Quantile Estimation Methods in Newsboy-Type Problems

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ABSTRACT

The newsboy problem solutions under the conditions of risk and of uncertainty about demand are well known. The former is the case where the distribution of demand is known or estimated, and the latter is the case where it is not known but the range of demand is given.

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I. <u>INTRODUCTION</u>

The newsboy problem was described by Morse and Kimball early in 1950 [1]. This problem has been presented in the literature under a variety of names, including newsvendor problem, christmas tree problem, and high-fashion apparel problem. This kind of problem may be described in the following way: The newsboy has to decide how many papers to purchase for resale. If he buys too many papers he will incur loss due to left-over papers (little salvage cost). If, on the other hand, he buys too few, he should also incur an opportunity cost due to lost sales. To determine the optimal order quantity, it is necessary to balance two opposing costs. Solutions to the newsboy problem under both conditions of risk and uncertainty are well known. The determination or estimation of the demand distribution is critical in dealing with this problem. Optimal solutions to the newsboy problem which minimize expected cost require information about a particular quantile of the demand distributions. If the demand follows some known distribution the problem can be solved by decision making under risk, as is also the case when the key quantile is known. With only information concerning the range of demand, optimal solutions can be found by some procedure such as the minimax rule under uncertainty.

The purpose of this study may be summarized as follows. We are concerned with decision policies during a period when

demand data is being acquired at the rate of one observation per decision period, starting with no data except an estimate of the maximum demand. To find the optimal expected value decision rule, five candidate quantile estimators will be proposed on the basis of order statistics. They will be evaluated by comparing their performance with each other and with the well-known minimax rule under uncertainty. An estimator which has the smallest relative cost deviation to the risk cost solution (ideal case) will be proposed as optimal. We shall also examine the switching time to change from the uncertainty rule to applying the proposed estimator.

In Chapter II, we shall review the newsboy problem solutions under conditions of both risk and uncertainty, and then suggest alternative approaches for demand estimation, and for the switching period from uncertainty to risk. For estimation, five quantile estimators will be proposed in Chapter III. In Chapter IV, candidate quantile estimators will be evaluated by simulation. The performance of the estimators, and candidate switching periods will be discussed in Chapter V. Conclusions and recommendations, for further work, will be given in the final chapter.

II. STATEMENT OF THE PROBLEM

In this section we shall briefly describe the well-known newsboy problem, illustrating its optimal solutions under both risk and uncertainty with respect to demand. Then we shall describe the quantile estimation problem we wish to solve.

A. THE NEWSBOY PROBLEM

The problem may be generalized in the following way. The decision maker selects a number (the newsboy's order quantity) but the actual demand (the number of papers his customers will want to purchase) may have a randomly determined value. If the decision maker's number is greater than this he pays a cost proportional to the difference, or he pays a cost proportional to the amount by which his number is smaller than this. His cost is zero only if his number is the same as the actual demand.

Let demand D be a continuous random variable and let quantity S be the number the decision maker selects to have on hand. The newsboy cost equation may be formulated as a two-piece continuous linear function in the following manner.

$$C(S) = \begin{cases} C_s(S - D) & \text{if} \quad 0 \le D \le S \\ \\ C_o(D - S) & \text{if} \quad D > S, \end{cases} \qquad (1)$$

where C_s is unit cost of surplus when S exceeds D, and C_o is unit cost of outage or shortage when D exceeds S. The cost function is shown in Figure 1.

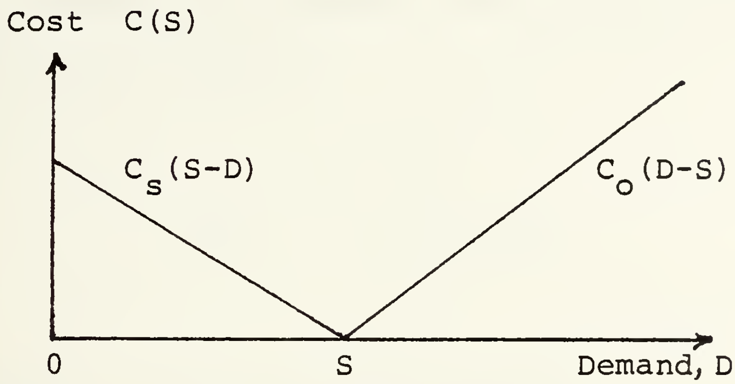

Figure 1. The Newsboy Cost Function

B. THE NEWSBOY PROBLEM SOLUTIONS UNDER RISK

For our case where demand is a continuous random variable D with probability density function f(D), and S is a decision variable whose value the decision maker is trying to find over D, the expected cost may be represented as:

$$E[C(S)] = C_s \int_0^S (S - D)f(D)dD + C_o \int_S^\infty (D - S)f(D)dD. \tag{2}$$

We wish to find quantity S, which minimizes the expected cost. The optimum (minimum in this case) value of S may be found by differentiating the expected cost function (2) with respect to S, setting the derivative to zero, and solving for the optimum value S*:

$$\frac{dE[C(S)]}{dS} = C_s \int_0^s f(D)\, dD - C_o \int_s^\infty f(D)\, dD \overset{\text{set}}{=} 0. \quad (3)$$

From this we obtain the optimal value S* using the cumulative distribution function

$$F(X) = \int_0^X f(D)\, dD,$$

and the result is the well-known result,

$$F(S^*) = \frac{C_o}{C_s + C_o} . \qquad (4)$$

Differentiating Equation (3) with respect to S to get the second derivative at S*,

$$\left.\frac{d^2E[C(S)]}{dS^2}\right|_{S=S^*} = (C_s + C_o)\ f(S^*) .$$

Hence, S* is the minimum expected cost solution since $(C_s + C_o)f(S^*)$ is positive.

To summarize, the optimal solution S* to minimize the expected cost is the $[C_o/(C_s + C_o)]$th quantile of the demand distribution, as shown in Figure 2, and the crucial information we need about the distribution is that quantile.

C. THE NEWSBOY PROBLEM SOLUTIONS UNDER UNCERTAINTY

The decision maker may be confronted with a newsboy problem without knowing or being able to estimate the

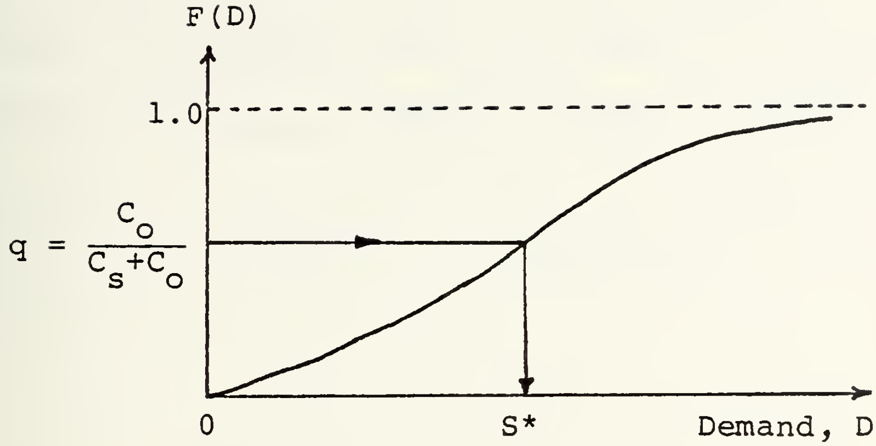

Figure 2. Minimum Expected Cost Solution

probability distribution of demand. The problem of deciding on S in this situation is called a decision under uncertainty. The decision maker may be able to estimate an upper bound for the demand value. In this case we say that all we know about demand is that demand D is such that $0 \leq D \leq D_{max}$.

Three well known approaches to decision making under uncertainty are Laplace solution under assumption of a uniform distribution of demand, minimax cost solutions and minimax regret solutions [2]. For the newsboy problem, these all lead to the same rule to find the optimal solution. We shall discuss only the minimax cost approach. This approach is to choose S so that the worst possible cost will be minimized, that is, to minimize maximum cost. The maximum cost occurs at D = 0 or at $D = D_{max}$, depending on the cost coefficients. This can be seen from Figure 1. If maximum cost occurs at D = 0, we would want to reduce S

while, if maximum cost occurs at $D = D_{max}$, we would want to increase S to reduce cost. We can minimize the maximum cost when S is chosen so that the cost at D = 0 is equal to the cost at $D = D_{max}$. Equating these, we obtain:

$$C_s S^* = C_o(D_{max} - S^*)$$

or

$$S^* = \left(\frac{C_o}{C_s + C_o}\right) D_{max} . \tag{5}$$

So far, we have introduced the minimum expected cost solution under both risk and uncertainty conditions. The minimax approach is tied to our initial estimate of D_{max}, and thus the optimal value S* from the minimax rule will be in error to the extent that our estimation of D_{max} is in error. This error will continue over many decision periods unless our estimate of D_{max} is revised.

D. QUANTILE ESTIMATION AND A DECISION RULE SWITCHING POINT FOR THE NEWSBOY PROBLEM

We are interested in finding a quantile estimator which gives better results than the minimax approach under conditions of limited demand data, and also in finding the switching period which is the time at which we switch from the minimax rule to the expected cost rule using the quantile estimator. The purposes of this study are to find the "best"

quantile estimator among five candidate estimators, and to obtain information about the switching period.

We recall that we wish to find the quantile of the cumulative distribution function, F(D). The optimal S* is found by an inverse mapping of the $[C_o/(C_s + C_o)]$th quantile of F(D) into demand D under risk condition. Let $q = C_o/(C_s + C_o)$, then we have:

$$S^* = F^{-1}(q) .$$

Although we don't know the demand distribution, each successive decision period provides another observation from the demand distribution, and we will assume that an observation of demand is indeed obtained each period. As we acquire more data we might be able to estimate $\hat{S}^* = \hat{F}^{-1}(q)$ as a function of the given data. Two kinds of approachs are:

1. a parameteric approach to hypothesizing the form of demand distribution,
2. a non-parametric approach to find the quantile.

For the first case, the normal, and exponential distributions have been found to be of considerable value in describing demand distributions. The normal distribution has been found to describe many demand functions at the factory level; the exponential, at the wholesale and retail levels [3]. Of course, these distributions should not be automatically applied to any demand distribution. Statistical

tests should establish the basis for any standard assumption concerning a demand function.

We are directing our attention to the case where the expected value solution is not applicable because of lack of available information. Further, if we had enough information for a fit of a distribution, we should have a good estimate of the quantile we seek. Hence, the second case, the non-parametric approach, will be considered to find the optimal S* and switching period.

III. DEVELOPMENT OF THE ESTIMATORS ON THE BASIS OF ORDER STATISTICS

As mentioned in the previous chapter, we are subject to find the optimum (minimum cost solution) of a newsboy problem without knowing the q-th quantile of the demand distribution. Consequently a non-parametric or distribution-free approach is suggested to find the solution. In this chapter, we will introduce the notion of order statistics as a basis for estimating this quantile, and suggest five candidate quantile estimators which may be used in newsboy solutions.

A. ESTIMATORS USING ORDER STATISTICS

The non-parametric approach we shall follow is on the basis of order statistics and although it is applicalbe to both continuous and discrete random variables, we shall direct our attention to the continuous case. We are interested in the case where the population is not known, but we have a random sample of size n from its unknown cumulative distribution function F(X). Let $X_1, X_2, \ldots, X_n$ be a random sample. Then $X(1) \leq X(2) \leq \ldots \leq X(n)$, where the X(i) are arranged in order of increasing magnitudes, and are defined to be the order statistics. Let X_q be the q-th quantile of the population. The estimate of X_q can be expressed as a function of $X_1, X_2, \ldots, X_n$, i.e.,

$$\text{Estimate of } X_q = f(X_1, X_2, \ldots, X_n) .$$

We shall consider now a function $F_n(x)$ which is constructed from the sample values of $X_1, X_2, \ldots, X_n$. The function $F_n(x)$ is called empirical cumulative distribution function. Let $x_1, x_2, \ldots, x_n$ denote the observed values of $X_1, X_2, \ldots, X_n$. Then for each number $x (-\infty < x < \infty)$, the value of $F_n(x)$ is defined to be the proportion of observed values in the sample which are less than or equal to x. It follows from the Glivenko-Cantelli Theorem that, as n approaches infinity the empirical cumulative distribution function $F_n(x)$ converges uniformly to $F(x)$. Since we are concerned with an unknown cumulative distribution function $F(x)$ which is continuous, some type of smoothed curve of $F_n(x)$ might yield a reasonable estimator of $F(x)$. Let q be the $[C_o/(C_s + C_o)]$th quantile, where we wish to find the estimate of X_q so that $F(\text{estimate of } X_q) = q$. This is represented in Figure 3. Now, we are interested in the estimator which gives us minimum cost, but we have no information about it yet. Let us turn our attention to the expected cost as the function of the estimate of X_q. Let perturbation be the difference between the estimate of X_q and true value of X_q. Then the expected cost of perturbation is proportional to the variance of the given unbiased estimator [4]. The properties of unbiasedness

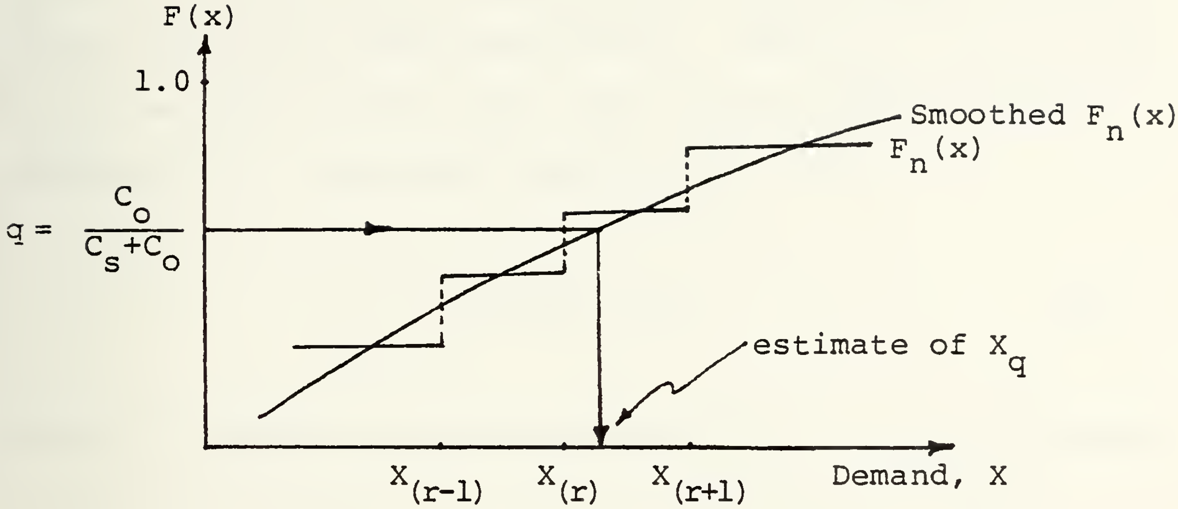

Figure 3. Estimate of X_q on the Smoothed Curve of Empirical CDF, $F_n(x)$

and minimum variance should be considered for selecting estimators.

B. RATIONALE FOR THE ESTIMATORS

In this section, we shall suggest five different quantile estimators on the basis of order statistics. In a statistical evaluation of these estimators, it is desirable to use the uniform (0,1) distribution, since the expectation and variance of its order statistics are simple and exact. Investigating these properties of order statistics using other distributions is not easy. If we have a distribution similar to the uniform, or if we have a large number of observations, some approximations may be possible. Three points which are near each other can be assumed to be linear and some estimator using order statistics would be approximately unbiased.

We shall examine the properties of the order statistics from the uniform (0,1) distribution, and we shall consider three neighboring order statistics. Let $X_{(r-1)}$, $X_{(r)}$, and $X_{(r+1)}$ be the (r-1)th, r-th, and (r+1)th order statistics respectively. Define the value r as:

$$r = [nq + 0.5] , \tag{6}$$

where [X] denotes the largest integer of X. Let us consider an estimate of X_q as the linear combination of those;

$$(\text{estimate of } X_q) = aX_{(r-1)} + bX_{(r)} + cX_{(r+1)},$$

where a, b, and c are nonnegative and sum to unity. There can be many possible ways to take estimators, but we shall examine typical ones considering the unbiasedness and the magnitude of variance.

1. <u>The First Candidate Estimator:</u> $\hat{X}_q$

First of all, the estimator should be unbiased, i.e., $E[\hat{X}_q] = X_q$. If we take the r-th order statistic, $X_{(r)}$ as the first candidate estimator, i.e.,

$$\hat{X}_q = X_{(r)} , \tag{7}$$

this is an unbiased estimator since $E[X_{(r)}] = r/(n+1)$ for the r-th order statistic. The coefficients are a = c = 0 and

b = 1 in this case. The first candidate estimator is represented in Figure 4.

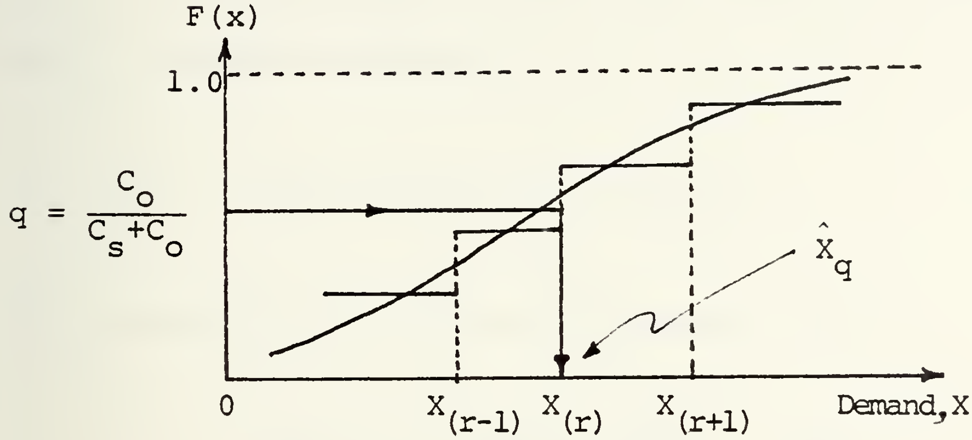

Figure 4. The First Candidate Estimator, $\hat{X}_q$

2. <u>The Second and Third Candidate Estimators:</u> $\tilde{X}_q$, $\tilde{\tilde{X}}_q$

We shall propose (r-1)th and (r+1)th order statistics as the second and third candidate estimators:

$$\tilde{X}_q = X_{(r-1)} , \tag{8}$$

and

$$\tilde{\tilde{X}}_q = X_{(r+1)} . \tag{9}$$

Even though these are not unbiased estimators, we are not certain which one gives us better performance since we are considering the properties on the uniform distribution and there is some round up error for taking the value r.

3. <u>The Fourth Candidate Estimator:</u> $\bar{X}_q$

We shall propose an equally weighted linear combination of the (r-1)th and (r+1)th order statistics as the fourth candidate estimator:

$$\bar{X}_q = \frac{1}{2}[X_{(r-1)} + X_{(r+1)}]. \qquad (10)$$

This estimator is an unbiased estimator, since

$$E[\bar{X}_q] = \frac{1}{2}[\frac{r-1}{n+1} + \frac{r+1}{n+1}] = \frac{r}{n+1},$$

and has the minimum variance using

$$\text{cov}[X_{(r)}, X_{(s)}] = \frac{r(n-s-1)}{(n+1)^2(n+2)}$$

of the uniform distribution [5]. The variance of this estimator is given as follows:

$$\text{Var}[\bar{X}_q] = \text{Var}[\frac{X_{(r-1)} + X_{(r+1)}}{2}] = \frac{(n-r)(2r-1)+(r-1)}{2(n+1)^2(n+2)}.$$

This estimator is represented in Figure 5.

4. <u>The Fifth Candidate Estimator:</u> $\bar{\bar{X}}_q$

We shall propose an equally weighted linear combination of the (r-1)th, r-th, and (r+1)th order statistics as the fifth candidate estimator, or

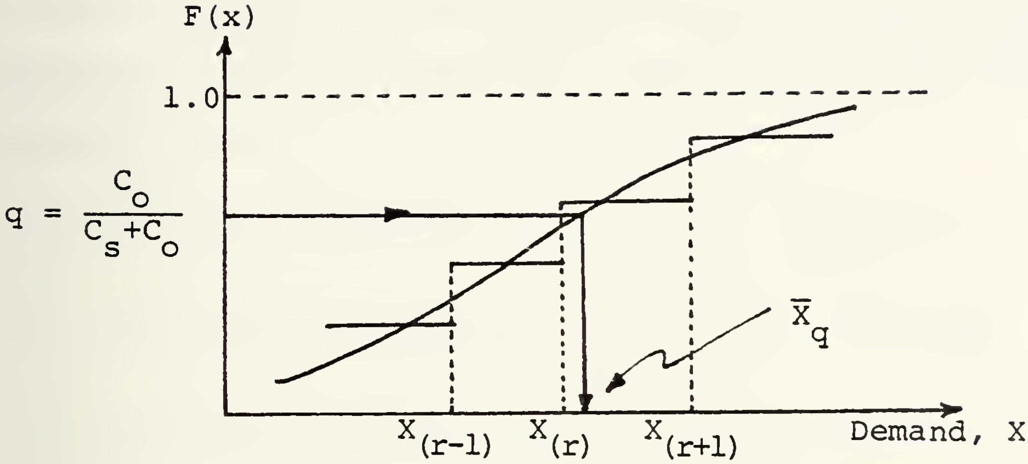

Figure 5. The Fourth Candidate Estimator, $\bar{X}_q$

$$\bar{\bar{X}}_q = \frac{1}{3}[X_{(r-1)} + X_{(r)} + X_{(r+1)}] . \tag{11}$$

This is also an unbiased estimator and the variance of this estimator is given by:

$$\text{Var } [\bar{\bar{X}}_q] = \frac{(n-r)(9r-4) + (5r-4)}{9(n+1)^2(n+2)}$$

The difference in variance between the fourth estimator and the fifth estimator is equal to $\frac{1}{18(n+1)(n+2)}$. When n = 1 and n = 10 this difference is approximately 0.09 and 4×10^{-4}, respectively. The difference converges rapidly to zero as n increases. Since the quantity difference may be assumed to be negligible and the variance is only concerned with uniform distribution, it is hard to say which estimator gives us better performance.

In summary, although there may be many possible candidate estimators depending upon different forms of linear combination, it may be reasonable to examine five previously proposed estimators. They can be shown with properties as:

1) The First Estimator: $\hat{X}_q = X_{(r)}$ (unbiased)

2) The Second Estimator: $\tilde{X}_q = X_{(r-1)}$ (biased)

3) The Third Estimator: $\tilde{\tilde{X}}_q = X_{(r+1)}$ (biased)

4) The Fourth Estimator: $\bar{X}_q = \frac{1}{2}[X_{(r-1)} + X_{(r+1)}]$ (unbiased min. variance)

5) The Fifth Estimator: $\bar{\bar{X}}_q = \frac{1}{3}[X_{(r-1)} + X_{(r)} + X_{(r+1)}]$ (unbiased).

Having proposed these quantile estimators, it is of interest to see how well they perform in the context of newsboy decision making, under cost criteria. This is the subject of the next chapter.

IV. EVALUATION OF THE CANDIDATE ESTIMATORS

In this chapter, the five candidate estimators will be tested and investigated for their performance in the newsboy problem for several demand distributions. As performance measures, two kinds of costs, namely an average cumulative cost and a mixed cost which we shall introduce later on, will be obtained during the simulation and then the performance of the candidate estimator will be evaluated using the relative cost deviation to the ideal cost.

A. DESIGN FOR SIMULATION

A newsboy simulation will consist of 50 replications, each of which simulates 50 decision periods for a given demand distribution. The quantile value ($q = C_o/(C_s + C_o)$) for each distribution is varied from 0.1 to 0.9 in increments of 0.2.

To find the value r and consequently to find the estimate of X_q, some modification of the method of determining r is necessary during the first and the last few periods, since there is an insufficient amount of data to apply the candidate estimators. For example, if the value r for finding the estimate is equal to zero, then $X_{(1)}$ will be chosen as the estimate of $\hat{X}_q$, and if r = n, then the estimate of $\tilde{\tilde{X}}_q$ will be $X_{(n)}$ and so forth.

For each period, the average cumulative cost and the mixed cost are computed for seven different cases which are

represented as follows:

1) Risk case (Ideal solution),

2) Uncertainty case using the minimax rule,

3) A case using the estimator $\hat{X}_q$,

4) A case using the estimator $\tilde{X}_q$,

5) A case using the estimator $\tilde{\tilde{X}}_q$,

6) A case using the estimator $\bar{X}_q$, and

7) A case using the estimator $\bar{\bar{X}}_q$.

The cost for each period is averaged over 50 replications, and then the average cumulative cost at period k is computed by cumulating the average cost from the first period to k-th period. The mixed cost at k-th period is given by summing the average cost using the minimax rule up to (k-1)th period and the average cost using an estimator from the k-th period to the last period.

B. DISTRIBUTION AND PARAMETERS

As mentioned previously, one source has suggested that the normal distribution may be considered as the demand distribution at the factory level and the exponential distribution may be considered as the demand distribution at the wholesale and retail levels, and so on. Five characterized forms of demand distributions including symmetric, right-skewed,

and left-skewed will be selected to test the performance of the candidate estimators for generality, and are shown in Figure 6.

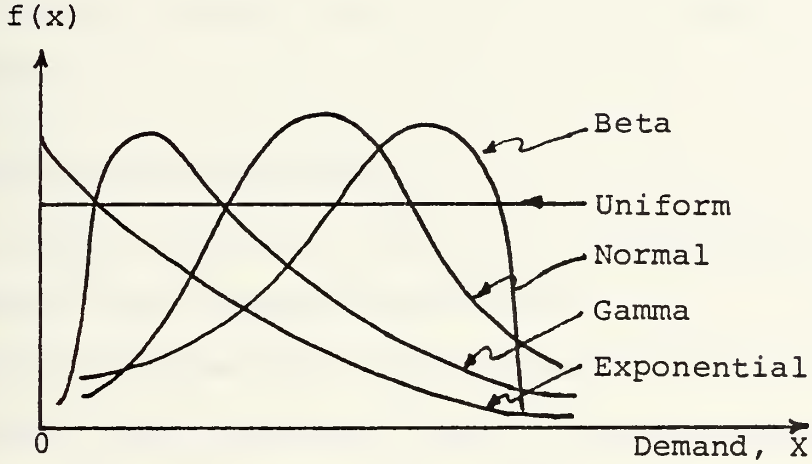

Figure 6. Characterized Forms of Demand Distributions

Demand distributions used in the simulation and their parameters are shown in Table I. The parameters for the normal distribution were selected so that negative demand should not be generated.

Table I. Distributions and Their Parameters

Distribution	Parameters
Uniform	The Range over [0,30]
Normal	Mean 35 and Variance 100
Exponential	Mean 20
Gamma	$\alpha = 2$ and $\lambda = 0.1$
Beta	A = 15 and B = 5

For each demand distribution, the 99.9th percentile of demand except uniform distribution will be adopted as D_{max} to apply the minimax rule under uncertainty. Quantile values $q = C_o/(C_s + C_o)$ will be 0.1, 0.3, 0.5, 0.7, and 0.9 for each distribution.

C. ALGORITHM FOR SIMULATION

The program steps are ordered in the following way:

1. Define a new demand distribution.
2. Assign the quantile value and maximum demand.
3. Compute the optimum in cases of uncertainty and risk.
4. Generate three random deviates from the specified demand distribution, representing demand for the first three periods.
5. Obtain the order statistics.
6. Compute the estimates using the candidate estimators.
7. Generate a random deviate for the specified demand distribution for the next period, compute the cost for the period and save it.
8. If the period is beyond the final period, go to Step 9; otherwise go to Step 5.
9. If the number of replications is greater than the maximum, go to Step 10; Otherwise go to Step 4.
10. Find the average cost for each period over total replications.
11. Compute the average cumulative cost, the mixed cost, and their relative cost deviation to the risk cost. Then tabulate the results.

12. If quantile value is greater than 0.9, go to Step 13; otherwise go to Step 2.

13. If we want to try another demand distribution go to Step 1; otherwise stop.

D. DISCUSSION OF THE SIMULATION RESULTS

In this section, the simulation results will be displayed and evaluated. As mentioned previously, the simulation run was performed over 50 replications, each of which consisted of 50 consecutive demand periods. From these trials, the average cumulative costs and the mixed costs were computed for various estimators.

Table II shows the average cumulative cost for 50 periods for the given quantile values and specified demand distribution. Errors in estimating D_{max} were not considered. This performance of quantile estimation may be compared directly with "best case" minimax results. Within a distribution we may conclude one estimator is better than the other but it is not easy to conclude for all distributions using Table II.

Now, a measure of effectiveness (MOE) to evaluate the performance of the estimators for all distributions is needed. Let us define the relative cost deviation of an estimator as follows:

$$\begin{array}{l}\text{Relative cost deviation} \\ \text{(to the risk cost)}\end{array} = \frac{\text{(cost using estimator) - (risk cost)}}{\text{(risk cost)}}$$

Table II

Average Cumulative Costs for 50 Periods and 50 Replications

Desc / Dist'n	Estimator	Quantile (q) 0.1	0.3	0.5	0.7	0.9
Uniform (0,3)	Risk	5.498	12.737	15.038	12.456	5.310
	Minimax	5.498	12.737	15.038	12.456	5.310
	$\hat{X}_q$	5.778	13.624	16.468	13.833	6.290
	$\tilde{X}_q$	5.846	14.095	17.397	15.117	7.702
	$\tilde{\tilde{X}}_q$	5.782	13.369	15.935	13.174	7.253
	$\bar{X}_q$	5.672	13.366	16.195	13.541	7.011
	$\bar{\bar{X}}_q$	5.687	13.376	16.205	13.580	6.604
Normal (35,100)	Risk	7.128	14.015	15.997	13.805	6.937
	Minimax	11.511	19.943	16.648	15.399	9.426
	$\hat{X}_q$	8.272	15.686	17.818	15.120	8.006
	$\tilde{X}_q$	8.790	16.870	19.473	16.892	9.417
	$\tilde{\tilde{X}}_q$	7.755	14.724	16.868	14.577	9.474
	$\bar{X}_q$	7.934	15.288	17.586	15.010	8.835
	$\bar{\bar{X}}_q$	7.944	15.232	17.502	14.980	8.329
Exponential (Mean = 20)	Risk	7.444	19.657	27.541	28.892	18.642
	Minimax	11.705	42.088	60.824	57.014	26.512
	$\hat{X}_q$	7.958	20.447	28.589	30.453	20.906
	$\tilde{X}_q$	7.949	20.425	28.853	30.434	21.114
	$\tilde{\tilde{X}}_q$	8.389	21.623	29.833	33.840	25.094
	$\bar{X}_q$	8.045	20.576	28.591	30.746	20.950
	$\bar{\bar{X}}_q$	8.046	20.568	28.547	30.447	20.072
Gamma (α = 2 λ = 0.1)	Risk	6.714	16.287	21.141	20.748	12.496
	Minimax	7.024	23.094	36.970	37.240	18.098
	$\hat{X}_q$	7.253	16.963	22.275	22.141	13.649
	$\tilde{X}_q$	7.303	17.180	22.496	22.442	14.138
	$\tilde{\tilde{X}}_q$	7.641	17.761	22.861	23.690	15.932
	$\bar{X}_q$	7.305	17.067	22.100	21.992	13.863
	$\bar{\bar{X}}_q$	7.297	17.043	22.098	21.964	13.299

Table II (Continued)

Desc / Dist'n	Estimator	Quantile (q) 0.1	0.3	0.5	0.7	0.9
Beta (A = 15 B = 5)	Risk	0.071	0.140	0.154	0.129	0.059
	Minimax	0.260	0.545	0.520	0.246	0.059
	$\hat{X}_q$	0.079	0.147	0.162	0.135	0.064
	$\tilde{X}_q$	0.079	0.150	0.167	0.141	0.070
	$\tilde{\tilde{X}}_q$	0.082	0.150	0.163	0.136	0.082
	$\bar{X}_q$	0.078	0.147	0.161	0.135	0.071
	$\bar{\bar{X}}_q$	0.078	0.147	0.161	0.135	0.067

This relative cost deviation may be used as an MOE to evaluate the performance of an estimator. The best performance was given by the risk solution during the simulation, an expected result with regard to theoretical considerations.

Table III exhibits the relative cost deviation for quantile values from 0.1 to 0.9 for each demand distribution investigated. As a special case, if we pursue the uniform demand distribution in Tables II and III, the risk and minimax approach would match each other because the risk cost solution X_q such that $F(X_q) = C_o/(C_s + C_o)$ is the same as the minimax cost solution given by $[C_o/(C_s + C_o)]D_{max}$ whenever the maximum demand D_{max} is determined exactly. If we read through the third column of overall distribution in Table III, an estimator, $\bar{X}_q$ shows the best performance among candidate estimators at quantile 0.1. Also, $\hat{X}_q$ did better at quantile 0.9. Excluding these two points, the estimator $\bar{\bar{X}}_q$ gave the best performance at all other quantiles. Within the uniform distribution the minimax approach shows the best performance (since it is identical to the risk solution), but it shows the worst performance in the other distribution. Similarly, we can say which estimator represents the best performance within a distribution by reading through row by row, and which estimator shows the best performance at a quantile by reading through column by column.

We are interested in the estimator which gives us the best performance for all quantiles over all distributions.

Table III

Relative Cost Deviation From The Average Cumulative Cost

Desc. / Dist'n	Estimator	Quantile (q) 0.1	0.3	0.5	0.7	0.9	Overall Quantile
Uniform (0.30)	Risk	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Minimax	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	$\hat{X}_q$	0.0509	0.0696	0.0949	0.1105	0.1845	0.5104
	$\tilde{X}_q$	0.0634	0.1066	0.1569	0.2136	0.4505	0.9909
	$\tilde{\tilde{X}}_q$	0.0517	0.0496	0.0597	0.0576	0.3658	0.5844
	$\bar{X}_q$	0.0318	0.0493	0.0770	0.0871	0.3203	0.5655
	$\bar{\bar{X}}_q$	0.0345	0.0502	0.0776	0.0902	0.2435	0.4960
Normal (35,100)	Risk	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Minimax	0.6149	0.4229	0.0405	0.1154	0.3589	1.5526
	$\hat{X}_q$	0.1605	0.1192	0.1136	0.1962	0.1542	0.6427
	$\tilde{X}_q$	0.2331	0.2037	0.2173	0.2236	0.3576	1.2353
	$\tilde{\tilde{X}}_q$	0.0879	0.0505	0.0544	0.0559	0.3658	0.6145
	$\bar{X}_q$	0.1130	0.0908	0.0994	0.0782	0.2736	0.6640
	$\bar{\bar{X}}_q$	0.1145	0.0868	0.0941	0.0850	0.2008	0.5812
Exponential (Mean = 20)	Risk	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Minimax	0.5724	1.1411	1.2085	0.9733	0.4222	4.3175
	$\hat{X}_q$	0.0690	0.0402	0.0381	0.0540	0.1215	0.3228
	$\tilde{X}_q$	0.0678	0.0391	0.0476	0.0603	0.1326	0.3474
	$\tilde{\tilde{X}}_q$	0.1269	0.1000	0.0832	0.1713	0.3461	0.8275
	$\bar{X}_q$	0.0806	0.0467	0.0381	0.0642	0.1238	0.3534
	$\bar{\bar{X}}_q$	0.0809	0.0463	0.0365	0.0538	0.0767	0.2942
Gamma ($\alpha = 2$ $\lambda = 0.1$)	Risk	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Minimax	0.0412	0.4180	0.7488	0.7949	0.4483	2.4562
	$\hat{X}_q$	0.0803	0.0415	0.0536	0.0671	0.0922	0.3347
	$\tilde{X}_q$	0.0878	0.0548	0.0641	0.0817	0.1314	0.4198
	$\tilde{\tilde{X}}_q$	0.1381	0.0905	0.0814	0.1418	0.2749	0.7267
	$\bar{X}_q$	0.0881	0.0479	0.0454	0.10600	0.1094	0.3508
	$\bar{\bar{X}}_q$	0.0869	0.0464	0.0453	0.0586	0.0642	0.3014

Table III (Continued)

Desc. / Dist'n	Estimator	Quantile (q) 0.1	0.3	0.5	0.7	0.9	Overall Quantile
Beta (A = 15 B = 5)	Risk	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Minimax	2.6322	2.9002	2.3714	0.9108	0.0085	8.8291
	$\hat{X}_q$	0.1023	0.0519	0.0549	0.0481	0.0864	0.3436
	$\tilde{X}_q$	0.1123	0.0753	0.0877	0.0952	0.1823	0.5528
	$\tilde{\tilde{X}}_q$	0.1481	0.0736	0.0558	0.0575	0.3852	0.7202
	$\bar{X}_q$	0.0916	0.0508	0.0459	0.0468	0.2019	0.4368
	$\bar{\bar{X}}_q$	0.0917	0.0481	0.0462	0.0451	0.1307	0.3618
Overall Dist'ns	Risk	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Minimax	3.8657	4.8822	4.3752	2.7944	1.2379	17.1554
	$\hat{X}_q$	0.4630	3.3224	0.3551	0.3749	0.6388	2.1542
	$\tilde{X}_q$	0.5644	0.4795	0.5736	0.6744	1.2543	3.5462
	$\tilde{\tilde{X}}_q$	0.5527	0.3642	0.3345	0.4841	1.7378	3.4733
	$\bar{X}_q$	0.4049	0.2855	0.3058	0.3453	1.0290	2.3705
	$\bar{\bar{X}}_q$	0.4085	0.2771	0.2997	0.3327	0.7159	2.0346

One way to compare is to note that the performance of an estimator overall quantiles over all distributions can be represented as the total sum of the relative cost deviations. Looking at the last column for overall quantiles over all distributions it may be concluded that the best performance (minimum sum) is given by the estimator $\bar{\bar{X}}_q = \frac{1}{3}[X_{(r-1)} + X_{(r)} + X_{(r+1)}]$ for all quantiles over all distributions as shown in Table III, because the minimum relative cost deviation is the closest to the risk cost (ideal cost solution). For specific quantiles, of course, occasionally other estimators performed better. In an application of the newsboy problem under initial uncertainty, one has information about the quantile q but often not about the form of the distribution. Accordingly, the information at the foot of Table III may provide information in selecting an estimator.

E. DISCUSSION OF SWITCHING RULES

When solving a newsboy problem repeatedly under uncertainty conditions we should probably use the minimax rule for the first period since there is no applicable amount of data to use in quantile estimation. Each successive decision period provides an additional piece of demand data which will contribute to quantile estimation. Therefore we are interested in the number of decision periods that should elapse before we apply the proposed estimator $\bar{\bar{X}}_q$ in place of the minimax rule. From the simulation results, Table IV shows the mixed cost with switching at period 25, the middle

Table IV

The Mixed Cost for the 25-th Period

Desc. / Dist.	Estimator	Quantile (q) 0.1	0.3	0.5	0.7	0.9
Uniform (0.30)	Risk	274.879	636.863	751.893	622.810	265.514
	Minimax	274.879	636.863	751.893	622.810	265.514
	$\hat{X}_q$	280.729	643.679	761.937	634.742	272.186
	$\tilde{X}_q$	282.951	648.889	766.665	640.032	277.903
	$\tilde{\tilde{X}}_q$	280.185	641.266	759.937	631.729	269.095
	$\bar{X}_q$	279.242	643.131	760.970	633.978	271.334
	$\bar{\bar{X}}_q$	279.595	643.109	761.187	634.082	271.463
Normal (35,100)	Risk	356.404	700.766	799.844	690.275	346.833
	Minimax	575.550	997.144	832.229	769.934	471.301
	$\hat{X}_q$	461.583	828.894	820.863	738.497	413.906
	$\tilde{X}_q$	477.707	835.620	825.044	742.866	420.868
	$\tilde{\tilde{X}}_q$	455.163	825.489	819.134	736.412	410.975
	$\bar{X}_q$	460.505	828.510	820.079	737.839	412.314
	$\bar{\bar{X}}_q$	460.535	828.517	820.258	737.909	412.595
Exponential (Mean = 20)	Risk	372.210	982.875	1377.038	1444.599	932.085
	Minimax	585.271	2104.407	3041.196	2850.687	1325.620
	$\hat{X}_q$	458.956	1453.137	2073.317	2028.197	1118.620
	$\tilde{X}_q$	458.897	1453.506	2075.067	2027.887	1115.982
	$\tilde{\tilde{X}}_q$	461.103	1456.723	2076.325	2036.363	1146.552
	$\bar{X}_q$	458.524	1453.449	2072.446	2027.401	1121.038
	$\bar{\bar{X}}_q$	458.556	1453.220	2072.413	2027.286	1119.067
Gamma ($\alpha = 2$, $\lambda = 0.1$)	Risk	335.684	814.349	1057.038	1037.387	624.824
	Minimax	351.179	1154.714	1848.499	1861.989	904.908
	$\hat{X}_q$	344.370	983.554	1414.183	1400.872	759.497
	$\tilde{X}_q$	344.799	985.234	1415.761	1402.143	759.026
	$\tilde{\tilde{X}}_q$	348.497	983.682	1410.905	1406.584	768.769
	$\bar{X}_q$	344.654	982.802	1410.740	1400.742	754.786
	$\bar{\bar{X}}_q$	344.504	982.942	1411.690	1400.479	755.480

Table IV (Continued)

Desc. / Dist.	Estimator	Quantile (q) 0.1	0.3	0.5	0.7	0.9
	Risk	3.573	6.993	7.699	6.437	2.948
Beta (A = 15 B = 5)	Minimax	12.988	27.275	26.003	12.299	2.973
	$\hat{X}_q$	7.691	15.796	15.779	9.190	3.021
	$\tilde{X}_q$	7.726	15.789	15.779	9.250	3.056
	$\approx{X}_q$	7.723	15.840	15.809	9.214	3.015
	$\underline{X}_q$	7.680	15.788	15.781	9.191	3.014
	$\bar{\bar{X}}_q$	7.678	15.789	15.779	9.190	3.015

period in our fifty-period simulation. The mixed costs here are the sum of costs for the first 24 periods given by the minimax rule and the costs given by an estimator from the 25-th to the last period. When we examine various distributions in Table IV the minimum mixed cost is obtained by the risk approach, which is to be expected, and the proposed estimator, $\bar{\bar{X}}_q$, appears to provide the minimum mixed cost among candidate estimators, although by a narrow margin.

Now, we wish to find the number of periods to retain the minimax rule before applying the proposed estimator $\bar{\bar{X}}_q$. Mixed costs were computed for various switching periods to find the period which gives the minimum mixed cost for each quantile of every distribution except uniform distribution, since the proposed estimator $\bar{\bar{X}}_q$ doesn't give better results than the minimax rule for the uniform distribution. Table V shows the best switching periods (minimax to estimator $\bar{\bar{X}}_q$) for the quantiles and distributions used in the simulation during 40 periods.

Table V

Best Switching Periods Using the Proposed Estimator $\bar{\bar{X}}_q$

Distribution	Quantile (q)				
	0.1	0.3	0.5	0.7	0.9
Normal	4	7	27	11	11
Exponential	6	4	4	4	4
Gamma	13	5	4	4	6
Beta	4	4	4	4	48

Looking at Table V, we see that the best switching period is 27 at quantile 0.5 of the normal distribution. This result is caused by the fact that the minimax rule behaves well around the median when demand has a symmetric distribution. The occurrence of switching period 48 at quantile 0.9 of the beta distribution can be explained by the fact that with the beta distribution used in the simulation, the optimal S* obtained by the minimax rule is close to the ideal S* at this quantile, i.e., the minimax S* at 0.9 is equal to 0.8767 and the ideal S* is 0.8660.

We shall suggest 6 periods, 4, 5, 6, 7, 11, and 13 as the candidates for the switching period for all quantiles over all distributions. Later periods are not included in the candidates since the estimator $\bar{\bar{X}}_q$ shows much better performance (with reference to the foot of Table III for nearly all quantiles over all distributions). Table IV shows the mixed costs obtained by the estimator $\bar{\bar{X}}_q$ for the 25th period. Similar tables (not shown) were computed for various other periods, and showed that the mixed costs increased as the period increased. We propose the relative mixed cost deviation for a candidate period using the estimator $\bar{\bar{X}}_q$ as an MOE. It may be reasonable to conclude that the period giving the minimum is the most preferable. Table VI shows the relative mixed cost deviation at each candidate period for all quantiles over all distributions using the estimator $\bar{\bar{X}}_q$.

Table VI

Relative Mixed Cost Deviation Using the Estimator $\bar{\bar{X}}_q$

Switching Period	Distribution	Estimator	Quantile (q)					
			0.1	0.3	0.5	0.7	0.9	Total
4	Uniform	$\bar{\bar{X}}_q$	.0345	.0502	.0776	.0902	.2435	.4960
	Normal		.1145	.0868	.0941	.0880	.2008	.5812
	Exponential		.0809	.0463	.0365	.0538	.0767	.2942
	Gamma		.0869	.0464	.0453	.0586	.0642	.3014
	Beta		.0917	.0481	.0462	.0451	.1307	.3618
	Overall		.4085	.2778	.3327	.3327	.7159	2.0346
5	Uniform	$\bar{\bar{X}}_q$	.0324	.0471	.0675	.0762	.1720	.3952
	Normal		.1201	.0868	.0751	.0673	.1462	.4955
	Exponential		.0708	.0548	.0561	.0669	.0851	.3337
	Gamma		.0576	.0422	.0555	.0713	.0708	.2971
	Beta		.1235	.1003	.0902	.0570	.0868	.4578
	Overall		.4044	.3312	.3444	.3387	.5606	1.9793
6	Uniform	$\bar{\bar{X}}_q$	.0317	.0457	.0626	.0693	.1229	.3322
	Normal		.1255	.0863	.0663	.0644	.1145	.4570
	Exponential		.0657	.0697	.0742	.0765	.0894	.3755
	Gamma		.0491	.0427	.0675	.0814	.0774	.3181
	Beta		.1715	.1591	.1400	.0790	.0608	.6104
	Overall		.4435	.4035	.4106	.3706	.4650	2.0932
7	Uniform	$\bar{\bar{X}}_q$	.0300	.0420	.0547	.0601	.1020	.2888
	Normal		.1311	.0846	.0533	.0572	.1084	.4346
	Exponential		.0738	.0924	.0980	.0944	.0987	.4573
	Gamma		.0423	.0487	.0754	.0924	.0847	.3435
	Beta		.2306	.2237	.1935	.0973	.0481	.7932
	Overall		.5078	.4914	.4749	.4014	.4419	2.3174

Table VI (Continued)

Switching Period	Distribution	Estimator	Quantile (q) 0.1	0.3	0.5	0.7	0.9	Total
11	Uniform	$\bar{\bar{X}}_q$	.0262	.0323	.0384	.0422	.0502	.1891
	Normal		.1727	.0952	.0368	.0526	.0988	.4561
	Exponential		.0859	.1641	.1779	.1594	.1292	.7165
	Gamma		.0194	.0797	.1253	.1384	.1041	.4669
	Beta		.4119	.4456	.3793	.1705	.0375	1.4448
	Overall		.7161	.8169	.7577	.5631	.4196	3.2734
13	Uniform	$\bar{\bar{X}}_q$	.0247	.0280	.0323	.0383	.0446	.1679
	Normal		.1916	.1047	.0322	.0544	.1070	.4899
	Exponential		.1057	.2066	.2228	.1932	.1413	.8696
	Gamma		.0910	.0930	.1502	.1627	.1158	.6127
	Beta		.5236	.5666	.4823	.2166	.0362	1.8253
	Overall		.9366	.9989	.9198	.6652	.4449	3.9654

The relative mixed cost deviation at each candidate period is shown in Table VII. The minimum deviation is given at period five and the deviation is increasing as the period increases from the fifth period. Using this result we may propose that the switching period from the minimax rule to applying the estimator $\bar{\bar{X}}_q$ should be the fifth period.

Table VII

The Relative Mixed Cost Deviation
At Each Candidate Period

Period	Cost Deviation
4	2.0346
5	1.9793 (minimum)
6	2.0932
7	2.3174
11	3.2734
13	3.9654

V. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

The minimax approach is a well known approach to decision making under uncertainty. For a newsboy-type problem, we developed five candidate estimators which could be used as data aggregates, and evaluated them by comparing with the minimax rule. These candidate estimators were tested for quantiles, 0.1, 0.3, 0.5, 0.7, and 0.9 of several distributions: uniform, normal (symmetric), exponential, gamma (right-skewed) and beta (left-skewed). Since all possible quantiles and other kinds of distributions were not covered, there may be some amount of loss of generality.

With these limitations, as discussed in Chapter IV, the estimator $\bar{\bar{X}}_q = \frac{1}{3}[X_{(r-1)} + X_{(r)} + X_{(r+1)}]$ developed on the basis of order statistics showed the best performance among those considered. The simulation results suggest that if a single decision procedure over all quantiles is needed, the estimator $\bar{\bar{X}}_q$ is recommended and the time for switching to this estimator should be at the fifth period. The proposed procedure may be summarized in the following way:

(1) Estimate D_{max}.

(2) Compute the quantile $q = C_o/(C_s + C_o)$.

(3) Apply the minimax rule $S^* = q \cdot D_{max}$ for the first four periods.

(4) Compute the value r such that $r = [nq + 0.5]$ from the fifth period where n is the period at which we wish

to estimate the demand and [X] denotes the largest integer of X.

(5) Compute the order statistics $X_{(r-1)}$, $X_{(r)}$, and $X_{(r+1)}$ and apply the proposed estimator $\bar{\bar{X}}_q$:

$$\bar{\bar{X}}_q = \frac{1}{3}[X_{(r-1)} + X_{(r)} + X_{(r+1)}] .$$

In this study, it was implicitly assumed that the value of demand for each period was obtainable even if D > S. For further study, the case when demand data greater than the quantity S is not available is worthy of investigation. Also, a non-parametric approach which can handle extreme quantiles (e.g., 0.01 or 0.99) and extremely skewed distributions could be investigated. The comparison between a parametric approach when the demand data are assumed to fit a probability distribution and non-parametric approach is also recommended. Finally, future simulation work on this problem should also record cost variances, so that a statistical analysis could be made of the simulation results.

APPENDIX

PROGRAM LISTING OF THE SIMULATION

```
                          APPENDIX

              PROGRAM LISTING OF THE SIMULATION
C
C
C---------------------------------------------------------------------
C                                                                    I
C                                                                    I
C  ..... THIS PROGRAM IS TO FIND THE OPTIMAL SOLUTION                I
C        (MINIMUM EXPECTED-COST) BY  FIVE CANDIDATE ESTIMATORS       I
C        ON THE BASIS OF ORDER STATISTICS                            I
C        IN COMPARISON WITH MINIMAX SOLUTION UNDER UNCERTAINTY       I
C        CONDITION. THE SIMULATION WILL BE PERFORMED FOR             I
C        QUANTILES 0.1,0.3,0.5,0.7,AND 0.9 OVER FIVE DIFFERENT       I
C        DISTRIBUTIONS ; UNIFORM,NORMAL,EXPONENTIAL,GAMMA,           I
C        AND BETA DISTRIBUTIONS.                                     I
C  ..... DEFINE THE VARIABLES AS ;                                   I
C        X --- TO SAVE THE CURRENT DEMAND                            I
C        XHAT1 --- TO SAVE THE ESTIMATE USING 1ST ESTIMATOR          I
C        XHAT2 --- TO SAVE THE ESTIMATE USING 2ND ESTIMATOR          I
C        XHAT3 --- TO SAVE THE ESTIMATE USING 3RD ESTIMATOR          I
C        XHAT4 --- TO SAVE THE ESTIMATE USING 4TH ESTIMATOR          I
C        XHAT5 --- TO SAVE THE ESTIMATE USING 5TH ESTIMATOR          I
C        COQ --- COST AT A PERIOD BY MINIMAX APPROACH                I
C        COR --- COST AT A PERIOD BY RISK APPROACH                   I
C        CXHA1 --- COST AT A PERIOD BY 1ST ESTIMATOR                 I
C        CXHA2 --- COST AT A PERIOD BY 2ND ESTIMATOR                 I
C        CXHA3 --- COST AT A PERIOD BY 3RD ESTIMATOR                 I
C        CXHA4 --- COST AT A PERIOD BY 4TH ESTIMATOR                 I
C        CXHA5 --- COST AT A PERIOD BY 5TH ESTIMATOR                 I
C        ACQU --- AVERAGE COST AT A PERIOD OVER MAX REPLICATION      I
C                 USING MINIMAX. SIMILARLY ACQR,ACXH1,ACXH2,ACXH3.   I
C        CACQL --- AVERAGE CUMULATIVE COST AT A PERIOD               I
C                  USING MINIMAX. SIMILARLY CACQR,CACXH1 ETC.        I
C                                                                    I
C                                                                    I
C---------------------------------------------------------------------
C
C
      INTEGER R,R1,RM
      DOUBLE PRECISION DSEED
      COMMON X(50,53)
      DIMENSION XHAT1(50,53),XHAT2(50,53),XHAT3(50,53)
      DIMENSION CQU(50,53),CQR(50,53),W(3)
      DIMENSION CXH1(50,53),CXH2(50,53),CXH3(50,53)
      DIMENSION  ACQU(53),ACQR(53),ACXH1(53),ACXH2(53),ACXH3(53)
      DIMENSION  CACQU(53),CACQR(53),CACXH1(53)
      DIMENSION CACXH2(53),CACXH3(53)
```

```
C
C-----------------------------------------------------------------
C  ..... FIVE DIFFERENT DISTRIBUTIONS WILL BE SIMULATED.     I
C-----------------------------------------------------------------
C
      DO 55555 KIND=1,5
C
      DO 1000 IS=1,3
C
C------------------------------------------------
C  ..... ASSIGN MAXIMUM DEMAND TO DMAX.      I
C------------------------------------------------
C
      IF(IS.NE.1) GO TO 2
      READ(5,4) DMAX
    4 FORMAT(F10.0)
      GO TO 5
    2 IF(IS.NE.2) GO TO 3
      READ(5,4) DMAX
      GO TO 5
    3 IF(IS.NE.3) GO TO 5
      READ(5,4) DMAX
    5 CONTINUE
C
C
C
C-----------------------------------------------------
C  ..... INITIALIZE THE FIXED SINGLE VARIABLES.   I
C-----------------------------------------------------
C
      XM=20
      MU=35
      SIG=10
      W(1)=0.
      MAXREP=50
      MAXP=53
      A=2.
      B=10.
      EPS=0.001
      CO=0-.4
C
C-------------------------------------------------------------------
C  ..... CLEAR ARRAY VARIABLES FOR ALL PERIODS AND REPLICATIONS.I
C-------------------------------------------------------------------
C
      DO 7 I=1,MAXREP
      DO 7 J=1,MAXP
      X(I,J)=0.
```

```
      XHAT1(I,J)=0.
      XHAT2(I,J)=0.
      XHAT3(I,J)=0.
      XHAT4(I,J)=0.
      XHAT5(I,J)=0.
      CQU(I,J)=0.
      CQR(I,J)=0.
      CXH1(I,J)=0.
      CXH2(I,J)=0.
      CXH3(I,J)=0.
      CXH4(I,J)=0.
      CXH5(I,J)=0.
    7 CONTINUE
      DO 8 I=1,MAXP
      ACQU(I)=0.
      ACQR(I)=0.
      ACXH1(I)=0.
      ACXH2(I)=0.
      ACXH3(I)=0.
      ACXH4(I)=0.
      ACXH5(I)=0.
      CACQU(I)=0.
      CACQR(I)=0.
      CACXH1(I)=0.
      CACXH2(I)=0.
      CACXH3(I)=0.
      CACXH4(I)=0.
      CACXH5(I)=0.
    8 CONTINUE
C
C-----------------------------------------------------------------------
C ..... ASSIGN QUANTILES TO Q AND FIND UNCERTAINTY SOLUTION.    I
C-----------------------------------------------------------------------
C
      DO 100 I=1,6
      CO=CO+0.8
      IF(I.EQ.6) CO=3.8
      CS=4.-CO
      Q=CO/(CO+CS)
      QU=Q*DMAX
C
C-----------------------------------------------------------------------
C ..... FIND RISK SOLUTION ACCORDING TO THE DISTRIBUTION FORM.  I
C-----------------------------------------------------------------------
C
      IF(KIND.EQ.1) QR=Q*30.
      IF(KIND.EQ.2) CALL NOR(Q,QR)
      IF(KIND.EQ.3) QR=-XM*ALOG(1.-Q)
```

```
      IF(KIND.EQ.4) CALL GAM(Q,QR)
C
C
C
      IF(KIND.EQ.5) CALL BET(Q,QR)
C
C-------------------------------------------------------------------
C ..... SIMULATION WILL BE PERFORMED UNTIL  MAXIMUM REPLICATION. I
C-------------------------------------------------------------------
C
      DO 200 J=1,MAXREP
      DSEED=1234577+J
      DO 201 J1=1,3
C
C-----------------------------------------------------------
C ..... GENERATE FIRST THREE RANDOM DEMAND DEVIATES     I
C       DEPENDING ON DISTRIBUTION FORM.                 I
C-----------------------------------------------------------
C
      IF(KIND.NE.1) GO TO 50
      CALL GGUBS(DSEED,1,U)
      X(J,J1)=U*30.
      GO TO 60
   50 IF(KIND.NE.2) GO TO 51
      CALL GGNML(DSEED,1,U)
      X(J,J1)=MU+U*SIG
      GO TO 60
   51 IF(KIND.NE.3) GO TO 52
      CALL GGEXN(DSEED,XM,1,U)
      X(J,J1)=U
      GO TO 60
   52 IF(KIND.NE.4) GO TO 53
      CALL GAMMA(GA,DSEED)
      X(J,J1)=GA
      GO TO 60
   53 CALL BETA(BE,DSEED)
      X(J,J1)=BE
   60 CONTINUE
C
  201 CONTINUE
C
C-----------------------------------------
C ..... GET THE ORDER STATISTICS  I
C-----------------------------------------
C
      CALL SORT(J,3)
      N=3
      R=N*Q+.5
```

```
C
      DO 300 K=4,MAXP
C
C-----------------------------------------------------------
C ..... MODIFICATION TO APPLY THE ESTIMATORS.  I
C-----------------------------------------------------------
C
      IF(R.GT.0) GO TO 301
      XH1=X(J,1)
      XH2=X(J,1)
      XH3=X(J,1)
      XH4=X(J,1)
      XH5=X(J,1)
      GO TO 304
  301 IF(R.GT.1) GO TO 302
      XH1=X(J,2)
      XH2=(X(J,1)+X(J,2))/2.
      XH3=(X(J,1)+X(J,2))/2.
      XH4=X(J,R)
      XH5=X(J,RM)
      GO TO 304
  302 IF(R.GE.MAXP) GO TO 303
      R1=R+1
      RM=R-1
      XH1=X(J,R1)
      XH2=(X(J,R1)+X(J,R)+X(J,RM))/3.
      XH3=(X(J,R1)+X(J,RM))/2.
C
C
C
      XH4=X(J,MAXP)
      XH5=X(J,MAXP)
      GO TO 304
  303 XH1=X(J,MAXP)
      MP1=MAXP-1
      XH2=(X(J,MP1)+X(J,MAXP))/2.
      XH3=(X(J,MP1)+X(J,MAXP))/2.
      XH4=X(J,ANXP)
      XH5=X(J,ANXP)
C
C------------------------------------------------------------------
C ..... FIND THE ESTIMATES USING CANDIDATE ESTIMATORS. I
C------------------------------------------------------------------
C
      XHAT1(J,K)=XH1
      XHAT2(J,K)=XH2
      XHAT3(J,K)=XH3
      XHAT4(J,K)=XH4
```

```
      XHAT5(J,K)=XH5
  304 CONTINUE
C
C
C
C-----------------------------------------------------------------
C  ..... GENERATE A RANDOM DEMAND DEVIATE DEPENDING UPON     I
C        DISTRIBUTION FORM AND FIND THE ESTIMATE.             I
C-----------------------------------------------------------------
C
      IF(KIND.NE.1) GO TO 70
      CALL GGUBS(DSEED,1,U)
      X(J,K)=U*30.
      GO TO 80
   70 IF(KIND.NE.2) GO TO 71
      CALL GGNML(DSEED,1,U)
      X(J,K)=MU+U*SIG
      GO TO 80
   71 IF(KIND.NE.3) GO TO 72
      CALL GGEXN(DSEED,XM,1,U)
      X(J,K)=U
      GO TO 80
   72 IF(KIND.NE.4) GO TO 73
      CALL GAMMA(GA,DSEED)
      X(J,K)=GA
      GO TO 80
   73 CALL BETA(BE,DSEED)
      X(J,K)=BE
   80 CONTINUE
      D=X(J,K)
C
C-----------------------------------------------------------------
C  ..... FIND THE COST AT EACH PERIOD UNTIL MAX PERIOD.    I
C-----------------------------------------------------------------
C
      IF(QU.GE.D) GO TO 305
      CQU(J,K)=CC*(D-QU)
      GO TO 306
  305 CQU(J,K)=CS*(QU-D)
  306 IF(QR.GE.D) GO TO 307
      CQR(J,K)=CC*(D-QR)
      GO TO 308
  307 CQR(J,K)=CS*(QR-D)
  308 IF(XH1.GE.D) GO TO 309
      CXH1(J,K)=CC*(D-XH1)
      GO TO 312
  309 CXH1(J,K)=CS*(XH1-D)
  312 IF(XH2.GE.D) GO TO 313
```

```
      CXH2(J,K)=CO*(D-XH2)
      GO TO 314
  313 CXH2(J,K)=CS*(XH2-D)
  314 IF(XH3.GE.D) GO TO 315
      CXH3(J,K)=CO*(D-XH3)
C
C
C
      GO TO 316
  315 CXH3(J,K)=CS*(XH3-D)
  316 CONTINUE
      IF(XH4.GE.D) GO TO 317
      CXH4(J,K)=CO*(D-XH4)
      GO TO 318
  317 CXH4(J,K)=CS*(XH4-D)
  318 IF(XH5.GE.D) GO TO 319
      CXH5(J,K)=CO*(D-XH5)
      GO TO 320
  319 CXH5(J,K)=CS*(XH5-D)
  320 CONTINUE
C
      CALL SORT(J,K)
      R=K*C+.5
  300 CONTINUE
  200 CONTINUE
C
C ------------------------------------------------------------------
C ..... PRINT OUT HEADING DEPENDING UPON DISTRIBUTION FORM.   I
C ------------------------------------------------------------------
C
      CALL PRT(KIND)
      WRITE(6,211)
  211 FORMAT(      40X,'      ORDER STATISTICS FOR RANDOM DEMAND    '///)
      WRITE(6,212) (X(J,K),K=1,MAXP)
  212 FORMAT(1X,10(F10.3,2X) )
      WRITE(6,250) Q,DMAX
  250 FORMAT(/// 5X,'AVG COST FOR EACH PERIOD OVER 50 REPLICATION',
     1' IN CASE OF QUANTILE = ',F6.3,' AND DMAX=',F7.3//
     210X,'PERIOD',5X,'U-AVG-COST',5X,'R-AVG-COST',5X,
     3'E1-AV-COST',5X,'E2-AV-COST',5X,'E3-AV-COST',
     45X,'E4-AV-COST',5X,'E5-AV-COST'//)
C
C-----------------------------------------------------
C ..... COMPUTE THE TOTAL COST AT EACH PERIOD.    I
C-----------------------------------------------------
C
      DO 120 K=1,MAXP
      TCGU=0.
```

```
      TCQR=C.
      TCXH1=0.
      TCXH2=C.
      TCXH3=C.
      TCXH4=0.
      TCXH5=C.
C
      DO 130 J=1,MAXREP
      TCQU=TCQU+CQU(J,K)
      TCQR=TCQR+CQR(J,K)
      TCXH1=TCXH1+CXH1(J,K)
      TCXH2=TCXH2+CXH2(J,K)
      TCXH3=TCXH3+CXH3(J,K)
      TCXH4=TCXH4+CXH4(J,K)
      TCXH5=TCXH5+CXH5(J,K)
  130 CONTINUE
C
C
C----------------------------------------------------
C ..... FIND THE AVERAGE COST AND PRINT OUT.  I
C----------------------------------------------------
C
      ACQU(K)=TCQU/MAXREP
      ACQR(K)=TCQR/MAXREP
      ACXH1(K)=TCXH1/MAXREP
      ACXH2(K)=TCXH2/MAXREP
      ACXH3(K)=TCXH3/MAXREP
      ACXH4(K)=TCXH4/MAXREP
      ACXH5(K)=TCXH5/MAXREP
      WRITE(6,141) K,ACQU(K),ACQR(K),ACXH1(K),ACXH2(K),ACXH3(K),
     C ACXH4(K),ACXH5(K)
C
C
C
  141 FORMAT(12X,I2,7X,6(F10.3,5X),F10.3)
  120 CONTINUE
C
      CALL PRT(KIND)
      WRITE(6,160) Q,DMAX
  160 FORMAT(     5X,'AVG CUMM COST UP TO I-TH PERIOD',
     1' IN CASE OF QUANTILE = ',F6.3,' AND DMAX=',F7.3//
     21X,'UP TO',3X,'CU-COST(CUTOCR)',3X,'CR-COST(CRTOCR)',
     33X,'C1-COST(C1TOCR)',3X,'C2-COST(C2TOCR)',
     43X,'C3-COST(C3TOCR)',3X,'C4-COST(C4TOCR)',3X,'C5-COST(C5TOCR)'//)
C
C------------------------------------------------------------
C ..... COMPUTE THE TOTAL AVERAGE COST UP TO A PERIOD.  I
C------------------------------------------------------------
```

```
C
      DO 150 K=1,MAXP
      TACQU=0.
      TACQR=0.
      TACXH1=0.
      TACXH2=0.
      TACXH3=0.
      TACXH4=0.
      TACXH5=0.
      CUROCR=0.
      CRTOCR=0.
      C1TOCR=0.
      C2TOCR=0.
      C3TOCR=0.
      C4TOCR=0.
      C5TOCR=0.
C
      IF(K.LE.3) GO TO 155
      DO 151 KC=4,K
      TACQU=TACQU+ACQU(KC)
      TACQR=TACQR+ACQR(KC)
      TACXH1=TACXH1+ACXH1(KC)
      TACXH2=TACXH2+ACXH2(KC)
      TACXH3=TACXH3+ACXH3(KC)
      TACXH4=TACXH4+ACXH4(KC)
      TACXH5=TACXH5+ACXH5(KC)
  151 CONTINUE
C
C ---------------------------------------------------------
C ..... FIND THE AVERAGE CUMMULATIVE COST.   I
C ---------------------------------------------------------
C
      K3=K-3
      CACQU(K)=TACQU/K3
      CACQR(K)=TACQR/K3
      CACXH1(K)=TACXH1/K3
      CACXH2(K)=TACXH2/K3
      CACXH3(K)=TACXH3/K3
      CACXH4(K)=TACXH4/K3
      CACXH5(K)=TACXH5/K3
      CUTOCR=(CACQU(K)-CACQR(K))/CACQR(K)
      CRTOCR=(CACQR(K)-CACQR(K))/CACQR(K)
      C1TOCR=(CACXH1(K)-CACQR(K))/CACQR(K)
      C2TOCR=(CACXH2(K)-CACQR(K))/CACQR(K)
      C3TOCR=(CACXH3(K)-CACQR(K))/CACQR(K)
      C4TOCR=(CACXH4(K)-CACQR(K))/CACQR(K)
      C5TOCR=(CACXH5(K)-CACQR(K))/CACQR(K)
C
```

```
C
C-----------------------------------------------------------
C .....  PRINT OUT AVERAGE  CUMULATIVE COST.  I
C-----------------------------------------------------------
  155 WRITE(6,161) K,CACQU(K),CUTOCR,CACQR(K),CRTOCR,
     1CACXH1(K),C1TOCR,CACXH2(K),C2TOCR,
     3CACXH3(K),C3TOCR,CACXH4(K),C4TOCR,CACXH5(K),C5TOCR
  161 FORMAT(1X,I2,4X,6(F8.3,'(',F6.4,')',2X),F8.3,'(',F6.4,')')
  150 CONTINUE
C
C
C
      CALL FFT(KIND)
      WRITE(6,165) Q,DMAX
  165 FORMAT(5X,'TOTAL OF AVG COST FOR FINDING TRANSITION PERIOD',
     3' IN CASE OF QUANTILE Q=',F6.3,'AND DMAX=',F7.3//
     15X,'(  UNCERTAINTY COST UP TO (I-1)TH PERIOD',
     2  ' + ESTIMATOR COST FROM I-TH PERIOD UNTIL 53RD PERIOD)'//
     41X,'WHEN'   ,5X,'UN-TR-COST(UN TO RI)',5X,'RI-TR-COST(RI TO RI)',
     55X,'E1-TR-COST(E1 TO RI)',5X,'E2-TR-COST(E2 TO RI)',
     65X,'E3-TR-COST(E3 TO RI)'//)
C
C-----------------------------------------------------------
C .....  FIND THE MIXED COST ( MINIMAX + EST. COST ).  I
C-----------------------------------------------------------
C
      DO 170 K=4,53
      UNCOST=0.
      RICOST=0.
      E1COST=0.
      E2COST=0.
      E3COST=0.
      E4COST=0.
      E5COST=0.
      KM=K-1
      DO 171 KA=1,KM
      UNCOST=UNCOST+ACQU(KA)
      RICOST=RICOST+ACQR(KA)
      E1COST=E1COST+ACQU(KA)
      E2COST=E2COST+ACQU(KA)
      E3COST=E3COST+ACQU(KA)
      E4COST=E4COST+ACQU(KA)
      E5COST=E5COST+ACQU(KA)
  171 CONTINUE
C
      DO 172 KB=K,53
      UNCOST=UNCOST+ACQU(KB)
      RICOST=RICOST+ACQR(KB)
```

```
      E1COST=E1CCST+ACXH1(KB)
      E2CCST=E2CCST+ACXH2(KB)
      E3CCST=E3CCST+ACXH3(KE)
      E4COST=E4CCST+ACXH4(KB)
      E5CCST=E5CCST+ACXH5(KE)
  172 CONTINLE
C
C-----------------------------------------------------------------
C ..... FIND THE RELATIVE DEVIATICN TO THE RISK SCLUTICN   I
C       FOR CUANTILES OVER DIFFERENT DISTRIBUTIONS.        I
C-----------------------------------------------------------------
C
      UNTRI=(UNCCST-RICOST)/RICOST
      RITRI=(RICCST-RICOST)/RICCST
      E1TRI=(E1CCST-RICOST)/RICOST
      E2TRI=(E2CCST-RICCST)/RICOST
      E3TRI=(E3CCST-RICCST)/RICOST
      E4TRI=(E4CCST-RICOST)/RICOST
      E5TRI=(E5CCST-RICOST)/RICCST
      WRITE(6,173) K,UNCCST,UNTRI,RICOST,RITRI,
     1E1COST,E1TRI,E2COST,E2TRI,E3COST,E3TRI,
     2E4COST,E4TRI,E5CCST,E5TRI
  173 FORMAT(1X,I2,4X,6(F8.3,'(',F6.4,')',2X),F8.3,'(',F6.4,')')
  170 CCNTINLE
C
  100 CONTINLE
C
C
 1000 CONTINLE
C
C
55555 CONTINLE
      STCP
      ENC



C--------------------------------------------------------------------
C ..... SUB PGM TC PRT CUT HEACING DEPENDING CN DISTRIBLTION.  I
C--------------------------------------------------------------------
C
      SUBRCUTINE PRT(KINC)
      IF(KINC.NE.1) GC TC 1C
      WRITE(6,1)
    1 FCRMAT(1H1,45X,
     1'UNIFCRM(C,30) DISTRIBLTICN CASE'//45X,32(1H*)////)
      RETURN
```

```
   10 IF(KIND.NE.2) GO TO 20
      WRITE(6,2)
    2 FORMAT(1H1,45X,
     1'NORMAL(35,100) DISTRIBUTION CASE'//45X,33(1H*)////)
      RETURN
   20 IF(KIND.NE.3) GO TO 30
      WRITE(6,3)
    3 FORMAT(1H1,45X,
     1'EXP(MEAN = 20) DISTRIBUTION CASE'//45X,33(1H*)////)
      RETURN
   30 IF(KIND.NE.4) GO TO 40
      WRITE(6,4)
    4 FORMAT(1H1,45X,
     1'GAMMA(R=2,LANDA=.1) DISTRIBUTION'//45X,33(1H*)////)
      RETURN
   40 WRITE(6,5)
    5 FORMAT(1H1,45X,
     1'BETA (A=15, B=5 )    DISTRIBUTION'//45X,33(1H*)////)
      RETURN
      END
C
C
C
C-----------------------------------------------------------
C ..... SUB PGM TO GET ORDER STATISTICS.  I
C-----------------------------------------------------------
C
      SUBROUTINE SORT(J,K)
      COMMON X(50,53)
      K1=K-1
      DO 20 KA=1,K1
      KB=KA+1
      DO 20 KS=KB,K
      IF(X(J,KA).LE.X(J,KS)) GO TO 20
      T=X(J,KA)
      X(J,KA)=X(J,KS)
      X(J,KS)=T
   20 CONTINUE
      RETURN
      END
C
C
C
C-----------------------------------------------------------
C ..... SUB PGM TO GENERATE GAMMA DEVIATE.  I
C-----------------------------------------------------------
C
      SUBROUTINE GAMMA(GA,DSEED)
```

```
      DOUBLE FRECISICN DSEEC
      DIMENSICN U(2)
      A=2.
      B=10.
      CALL GGUES(DSEED,2,U)
      UU=U(1)*U(2)
      GA=-B*ALCG(UU)
      RETURN
      END
C
C
C
C-----------------------------------------------------
C ..... SUB PGM TO GENERATE BETA  DEVIATE.   I
C-----------------------------------------------------
C
C
      SUBRCUTINE BETA(BE,DSEED)
      DOUBLE FRECISICN DSEEC
      DIMENSICN U(20)
      A=15.
      B=5.
      M=A
      N=B
      MPN=M+N
      CALL GGUES(DSEED,MPN,U)
      UAP=1.
      UBP=1.
      DO 10 I=1,M
      UAP=UAP*U(I)
   10 CONTINUE
      M1=M+1
      DO 20 I=M1,MPN
      UBP=UBP*U(I)
   20 CONTINUE
      XA=-ALCG(UAP)
      XB=-ALCG(UBP)
      BE=XA/(XA+XB)
      RETURN
      END
C
C
C
C-----------------------------------------------------
C ..... SUB PGM TC FIND RISK SCL OF NORMAL.   I
C-----------------------------------------------------
C
C
```

```
      SUBROUTINE NOR(Q,QR)
      MU=35
      SIG=10.
      EPS=0.001
     IF(ABS(Q-0.1).GE.EPS) GO TO 14
      ZQ=-1.282
      GO TO 19
   14 IF(ABS(Q-0.3).GE.EPS) GO TO 15
      ZQ=-0.524
      GO TO 19
   15 IF(ABS(Q-0.5).GE.EPS) GO TO 16
      ZQ=0.
      GO TO 19
   16 IF(ABS(Q-0.7).GE.EPS) GO TO 17
      ZQ=0.524
      GO TO 19
   17 IF(ABS(Q-0.9).GE.EPS) GO TO 18
      ZQ=1.282
      GO TO 19
   18 IF(ABS(Q-.95).GE.EPS) GO TO 19
      ZQ=1.645
   19 CONTINUE
      QR=MU+SIG*ZQ
      RETURN
      END
C
C
C
C-----------------------------------------------------------
C ..... SUB PGM TO FIND RISK SCL OF GAMMA.                 I
C-----------------------------------------------------------
C
      SUBROUTINE GAM(Q,QR)
      EPS=0.001
      IF(ABS(Q-0.1).GE.EPS) GO TO 14
      QR=5.32
      GO TO 19
   14 IF(ABS(Q-0.3).GE.EPS) GO TO 15
      QR=10.98
      GO TO 19
   15 IF(ABS(Q-0.5).GE.EPS) GO TO 16
      QR=16.79
      GO TO 19
   16 IF(ABS(Q-0.7).GE.EPS) GO TO 17
      QR=24.4
      GO TO 19
   17 IF(ABS(Q-0.9).GE.EPS) GO TO 18
      QR=38.9
```

```
      GO TO 19
   18 IF(ABS(Q-0.95).GE.EPS) GO TO 19
      QR=60.
   19 CONTINUE
      RETURN
      END
C
C
C
C------------------------------------------------------------------
C ..... SUB PGM TO FIND RISK SOL OF BETA.                        I
C------------------------------------------------------------------
C
      SUBROUTINE BET(Q,QR)
      EPS=0.001
      IF(ABS(Q-0.1).GE.EPS) GO TO 14
      QR=0.6224731
      GO TO 19
   14 IF(ABS(Q-0.3).GE.EPS) GO TO 15
      QR=0.7054668
      GO TO 19
   15 IF(ABS(Q-0.5).GE.EPS) GO TO 16
      QR=0.7584574
     GO TO 19
   16 IF(ABS(Q-0.7).GE.EPS) GO TO 17
      QR=0.8066888
      GO TO 19
   17 IF(ABS(Q-0.9).GE.EPS) GO TO 19
      QR=0.8660645
   19 CONTINUE
      RETURN
      END
C
```

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A study of alternative quantile estimation methods in newsboy-type problems.